

# Quantum criticality and correlations in the cuprate superconductors

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A description of the electronic correlations contained in the Hubbard model on the square-lattice perturbed by very weak three-dimensional uniaxial anisotropy in terms of the residual interactions of charge  $c$  fermions and spin-neutral composite two-spinon  $s1$  fermions is used to access further information on the origin of quantum critical behavior in the hole-doped cuprate superconductors. Excellent quantitative agreement with their anisotropic linear- $\omega$  one-electron scattering rate and normal-state linear- $T$  resistivity is achieved. Our results provide strong evidence that the normal-state linear- $T$  resistivity is indeed a manifestation of low-temperature scale-invariant physics.

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The interplay between quantum critical behavior [1–5] and the mechanism underlying the pairing state of the high-temperature superconductors [6–8] remains an enigmatic issue. The Hubbard model on the square lattice [6, 7, 9–11] perturbed by very weak three-dimensional uniaxial anisotropy provides the simplest realistic description of the role of correlations effects in the properties of the hole-doped cuprate superconductors. Recent experiments on these systems [3, 4, 12–19] impose new severe constraints on the mechanisms responsible for their unusual properties.

The virtual-electron pair quantum liquid (VEPQL) [11] describes the above toy model electronic correlations in terms of residual  $c$  -  $s1$  fermion interactions. Alike the Fermi-liquid quasi-particle momenta [20], those of the  $c$  and  $s1$  fermions are close to good quantum numbers [10, 11]. The results of Ref. [11] provide evidence that for a hole concentration domain the VEPQL short-range spin order coexists with a long-range  $d$ -wave superconducting order consistent with unconventional superconductivity being mediated by magnetic fluctuations [13]. The  $U(1)$  phase symmetry broken below  $T_c$  refers to the hidden  $U(1)$  symmetry recently found in Ref. [9]. Each *virtual-electron pair* configuration involves one  $c$  fermion pair of charge  $-2e$  and one spin-singlet two-spinon  $s1$  fermion whose spin-1/2 spinons are confined within it.

The magnitudes of the basic parameters appropriate to  $\text{YBa}_2\text{Cu}_3\text{O}_{6+\delta}$  (YBCO 123) and  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  (LSCO) used in this Letter are within the VEPQL scheme the effective interaction and transfer integral ratio  $U/4t \approx 1.525$  where  $t \approx 295$  meV and  $T = 0$  critical hole concentrations  $x_c \approx 0.05$  and  $x_* = 0.27$  for both such systems, lattice spacing  $a \approx 3.9$  Å, average separation between  $\text{CuCO}_2$  planes  $d_{\parallel} \approx 5.9$  Å, maximum  $s1$  fermion pairing energy per spinon  $\Delta_0 \approx 84$  meV, and coefficient  $C_{s1} = 1$  for YBCO 123 and  $a \approx 3.8$  Å,  $d_{\parallel} \approx 6.6$  Å,  $\Delta_0 \approx 42$  meV, and  $C_{s1} = 2$  for LSCO [11]. The VEPQL predictions achieve a good agreement with the cuprates universal properties [11] and those of their parent compounds [10] and consistency with the coexisting two-gap scenario [12]: A pseudogap  $2|\Delta| \approx (1 - x/x_*)2\Delta_0$

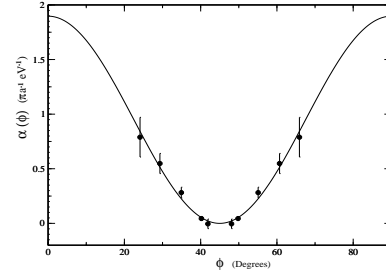


FIG. 1: The theoretical coefficient  $\alpha(\phi) = (\cos 2\phi)^2 / (x 64 x_* \sqrt{\pi x_{op}} t)$  (solid line) for  $x = 0.145$  and the LSCO parameters  $x_* = 0.27$  and  $x_{op} = 0.16$  together with the experimental points for the corresponding coefficient  $[\alpha_I(\phi) - \alpha_I(\pi/4)]$  of Fig. 4 (c) of Ref. [15].

and superconducting energy scale  $2|\Omega| \approx 4k_B T_c / (1 - [x_c/x_*](T_c/T_c^{max}))$  over the whole dome  $x \in (x_c, x_*)$ , where  $T_c \approx \gamma_d [(x - x_c)/(x_* - x_c)](1 - x/x_*)[\Delta_0/2k_B]$  and  $(1 - x_c/x_*) \geq \gamma_d \geq 1$ . Those are the maximum magnitudes of the spinon pairing energy and superconducting virtual-electron pairing energy, respectively.

In the ground state there is no one-to-one correspondence between a  $c$  fermion pair and a two-spinon  $s1$  fermion in that such objects may participate in several virtual-electron pairs. Specifically, the strong effective coupling of  $c$  fermion pairs whose hole momenta  $\vec{q}^h$  and  $-\vec{q}^h$  belong to an approximately circular  $c - sc$  line centered at  $-\vec{\pi} = -[\pi, \pi]$  results from interactions within virtual-electron pair configurations with a well-defined set of  $s1$  fermions whose two spinons momenta  $\pm\vec{q}$  belong to a uniquely defined  $s1 - sc$  line arc centered at  $\vec{0} = [0, 0]$ . A  $c - sc$  line has radius  $q^h = |\vec{q}^h| \in (q_{Fc}^h, q_{ec}^h)$  and  $c$  fermion energy  $|\epsilon_c(q^h)| \in (0, W_{ec})$  such that  $|\epsilon_c(q_{Fc}^h)| = 0$  and  $|\epsilon_c(q_{ec}^h)| = W_{ec} = 4\Delta_0/(1 - x_c/x_*)$ . For Fermi angles  $\phi \in (0, \pi/2)$  the corresponding  $s1 - sc$  line arc can be labelled either by its nodal momentum absolute value  $q_{arc}^N = q_{arc}^N(q^h) \in (q_{ec}^N, q_{Bs1}^N)$  or angular width  $2\phi_{arc} = \arcsin([q_{arc}^N - q_{ec}^N]/[q_{Bs1}^N - q_{ec}^N]) \in (0, \pi/2)$ . Here and above  $q_{ec}^N \approx q_{Bs1}^N - [\Delta_0/t]\{C_{s1}/[(x_* - x_c)\sqrt{(q_{Bs1}^N)^2 - (q_{ec}^N)^2}]\}$ ,  $q_{ec}^h \approx (\sqrt{1 + [\Delta_0/(x(x_* - x_c)\pi^2 t)]})q_{Fc}^h$ ,  $q_{Bs1}^N$  and  $q_{Bs1}^h$  are

nodal and anti-nodal momentum absolute values, respectively, belonging to the strongly anisotropic  $s1$  band boundary-line centered at  $\vec{0}$  [10], and  $q_{Fc}^h \approx \sqrt{x\pi}2$  refers to the isotropic  $c$  Fermi line centered at  $-\vec{\pi}$ . The energy needed for the  $c$  fermion strong effective coupling is supplied by the short-range spin correlations through the  $c$  -  $s1$  fermion interactions within each virtual-electron pair configuration. *Strong*  $c$  fermion effective coupling is that whose corresponding virtual-electron pair breaking under one-electron removal excitations gives rise to sharp-feature-line arcs centered at momenta  $\pm\vec{\pi} = \pm[\pi, \pi]$ . Those are in one-to-one correspondence to the  $s1$  -  $sc$ -line arcs of the virtual-electron pair  $s1$  fermion. Such sharp-feature-line arcs have angular range  $\phi \in (\pi/4 - \phi_{arc}, \pi/4 + \phi_{arc})$  and energy  $E \approx 2W_{ec}(1 - \sin 2\phi_{arc})$ . Hence they exist only for  $E < E_1(\phi) = 2W_{ec}(1 - |\cos 2\phi|)$ . The macroscopic condensate refers to  $c$  fermion pairs whose phases  $\theta = \theta_0 + \theta_1$  line up. The fluctuations of  $\theta_0$  and  $\theta_1$  become large for  $x \rightarrow x_c$  and  $x \rightarrow x_*$ , respectively. The dome  $x$  dependence of the critical temperature  $T_c$  is fully determined by the interplay of such fluctuations. A pseudogap state with short-range spin order and virtual-electron pair configurations without phase coherence occurs for temperatures  $T \in (T_c, T^*)$  where  $T^* \approx C_{s1}(1 - x/x_*)[\Delta_0/2k_B]$  is the pseudogap temperature. At  $T = 0$  a normal state emerges by application of a magnetic field aligned perpendicular to the planes of magnitude  $H \in (H_0, H_{c2})$  for  $x \in (x_0, x_{c2})$  and  $H \in (H_0, H^*)$  for  $x \in (x_{c2}, x_*)$ . The fields  $H_0$ ,  $H_{c2}$ , and  $H^*$  and the hole concentration  $x_0 < x_c$  are given in Ref. [11]. For  $x \in (x_0, x_{c1})$  the upper magnetic field  $H_{c2}(x)$  refers to the straight line plotted in Fig. 4 of that reference where  $x_0 \approx 0.013$  and  $x_{c1} = 1/8$ . However, for  $x \in (x_1, x_{c2})$  the actual  $H_{c2}(x)$  line may (or may not) slightly deviate to below the straight line plotted in that figure. If so, the hole concentration  $x_{c2} \approx 0.20$  may increase to  $\approx 0.21 - 0.22$ .

Fortunately, such a possible deviation does not change the physics discussed here.

The main goals of this Letter are: i) The study of the one-electron scattering rate and normal-state  $T$ -dependent resistivity within the VEPQL; ii) Contributing to the further understanding of the role of scale-invariant physics in the unusual scattering properties of the hole-doped cuprates. Our results refer to a range  $x \in (x_A, x_{c2})$  for which  $V_{Bs1}^\Delta/V_{Fc} \ll 1$ . Here  $x_A \approx x_*/2 = 0.135$  and the  $s1$  boundary line and  $c$  Fermi velocities read  $V_{Fc} \equiv V_c(\vec{q}_{Fc}^{hd}) \approx [\sqrt{x\pi}2/m_c^*]$  and  $V_{Bs1}^\Delta \equiv V_{s1}^\Delta(\vec{q}_{Bs1}^d) \approx [|\Delta|/\sqrt{2}]|\sin 2\phi|$ , respectively, where  $m_c^*$  is the  $c$  fermion mass. For  $x \in (x_{c2}, x_*)$  that inequality is also fulfilled but there emerge competing scattering processes difficult to describe in terms of  $c$  -  $s1$  fermion interactions. Elsewhere it is shown that the VEPQL predictions agree quantitatively with the distribution of the LSCO sharp photoemission spectral features of Figs. 3 and 4 of Ref. [18]. As predicted, they occur for energies  $E(\phi) < E_1(\phi)$  and the corresponding sharp-feature line arcs angular ranges agree with the theoretical magnitudes. This reveals experimental spectral signatures of the VEPQL virtual-electron pairing mechanism.

Here we start by using a Fermi's golden rule in terms of the  $c$  -  $s1$  fermion interactions to calculate for small  $\hbar\omega$  the one-electron inverse lifetime. Upon removal of one electron, two holes emerge in the  $s1$  and  $c$  bands, respectively. For low transfer energy  $\hbar\omega$  and small transfer momentum  $\vec{p}$  the  $c$  -  $s1$  fermion inelastic collisions conserve the doublicity  $d = \pm 1$ , which refers to one-electron excited states with the same energy and momentum but different electron velocity [10, 11]. Within such processes one  $s1$  fermion moves to the single hole in the  $s1$  band. One must then integrate over all particle-hole or hole-particle processes in the  $c$  fermion band that conserve energy and momentum. For low  $\hbar\omega$  and small  $\vec{p}$  the one-electron inverse lifetime can then be written as,

$$\frac{\hbar}{\tau_{el,d}} = 2\pi \int \frac{dq^h{}^2}{[2\pi]^2} |W_{c,s1}(\vec{q}^h, \vec{q}_{Bs1}^d; \vec{p})|^2 N_c(\vec{q}^h) N_c^h(\vec{q}^h + \vec{p}) N_{s1}^h(\vec{q}_{Bs1}^d - \vec{p}) \delta(\epsilon_c(\vec{q}^h + \vec{p}) - \epsilon_c(\vec{q}^h) + \epsilon_{s1}(\vec{q}_{Bs1}^d - \vec{p}) - \epsilon_{s1}(\vec{q}_{Bs1}^d)), \quad (1)$$

where  $d = \pm 1$ . The number of  $c$  fermions equals that of spin-up plus spin-down electrons, so that there is no additional factor 2 in this expression. The  $c$  and  $s1$  fermion energy dispersions and momentum distribution functions appearing here are introduced in Refs. [10, 11] and  $W_{c,s1}(\vec{q}^h, \vec{q}; \vec{p})$  is the matrix element of the  $c$  -  $s1$  fermion effective interaction between the initial and final states. It can be estimated for the hole concentration range  $x \in (x_A, x_{c2})$  for which  $r_\Delta = V_{Bs1}^\Delta/V_{Fc} \ll 1$ . Indeed, then the single heavy  $s1$  fermion hole plays mainly

the role of a scattering center and the  $c$  fermion holes that of scatterers and one can evaluate the matrix element absolute value  $|W_{c,s1}(\vec{q}^h, \vec{q}; \vec{p})|$  to zeroth order in  $V_{Bs1}^\Delta/V_{Fc} \ll 1$ . Provided that the small velocity  $V_{Bs1}^\Delta$  is accounted for in the physical quantities of expression (1) other than that matrix element, such a procedure leads to a good approximation for the one-electron inverse lifetime  $\hbar\omega$  dependence. For that  $x$  range we then find  $\lim_{\vec{p} \rightarrow 0} |W_{c,s1}(\vec{q}^h, \vec{q}; \vec{p})| \approx [\pi/4\rho_c(\vec{q}^h)]|\sin(\delta_1 - \delta_0)|$  where  $\vec{q}^h \approx \vec{q}_{Fc}^{hd}$ ,  $\vec{q} \approx \vec{q}_{Bs1}^d$ , the angular-momentum

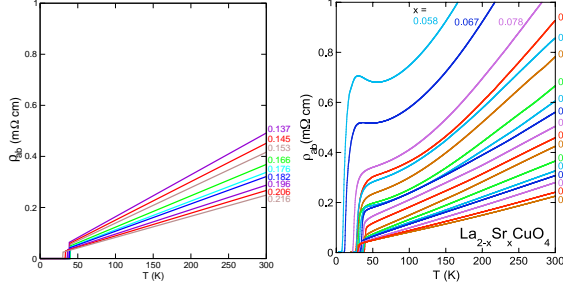


FIG. 2: (a) The  $T$  dependence of the resistivity  $\rho(T, 0) = \theta(T - T_c) \rho(T)$  with  $\rho(T)$  given in Eq. (4) for  $x \in (x_A, x_{c2})$  where  $x_A \approx 0.135$  and  $x_{c2} \approx 0.20 - 0.22$  and the parameter magnitudes for LSCO. (b) Corresponding experimental curves. Experimental curves figure from Ref. [21].

$l = 0, 1$  phase shifts read  $\delta_0 = \pi/2$  and  $\delta_1 = 2\phi$ , respectively, and the density of states  $\rho_c(\vec{q}^h) = m_c^*/2\pi\hbar^2$  of the present effective two-dimensional  $c$  fermion scattering problem is independent of  $x$ . Hence we arrive to  $\lim_{\vec{p} \rightarrow 0} |W_{c,s1}(\vec{q}^h, \vec{q}; \vec{p})| \approx [\pi^2 \hbar^2 / 2m_c^*] |\cos 2\phi| \approx \pi^3 x_* t |\cos 2\phi|$ . Fortunately, for  $x \in (x_A, x_{c2})$  the quantities contributing to (1) are independent of the doubly  $d = \pm 1$ , so that after some algebra we arrive to an inverse lifetime  $\hbar/\tau_{el} \approx \hbar\omega \pi \alpha_{\tau_{el}}$  and scattering rate  $\Gamma(\phi, \omega) = 1/[\tau_{el} V_F] \approx \hbar\omega \pi \alpha$ . Here  $V_F \approx V_{Fc} \approx [\sqrt{x\pi}/2m_c^*]$ ,  $\alpha_{\tau_{el}} = [\pi/16\sqrt{x x_{op}}](\cos 2\phi)^2$ , and  $\alpha = (\cos 2\phi)^2 / (x 64 x_* \sqrt{\pi x_{op}} t)$  where  $x_{op} = (x_* + x_c)/2 = 0.16$ . (We use units of lattice constant  $a = 1$ .) Such small- $\hbar\omega$  expressions are expected to remain valid for approximately  $\hbar\omega < E_1(\phi) = 2W_{ec}(1 - |\cos 2\phi|)$ . The factor  $(\cos 2\phi)^2$  also appears in the anisotropic component of the scattering rate studied in Ref. [19] for hole concentrations  $x > x_{c2}$ . For  $x = 0.145$  the use of the LSCO parameters leads to the theoretical coefficient  $\alpha(\phi)$  plotted in Fig. 1 (solid line) together with the experimental points of Fig. 4 (c) of Ref. [15] for  $[\alpha_I(\phi) - \alpha_I(\pi/4)]$ . (The very small  $\alpha_I(\pi/4)$  magnitude is related to processes that are not contained in the VEPQL.) An excellent quantitative agreement is obtained between  $\alpha(\phi)$  and the experimental points of  $[\alpha_I(\phi) - \alpha_I(\pi/4)]$ .

In the following we provide strong evidence from agreement between theory and experiments that the linear- $T$  resistivity is indeed a manifestation of normal-state scale-invariant physics. This requires that the  $T$ -dependence of the inverse relaxation lifetime derived for finite magnetic field,  $x \in (x_A, x_{c2})$ , and  $\hbar\omega \ll \pi k_B T$  by replacing  $\hbar\omega$  by  $\pi k_B T$  in the one-electron inverse lifetime  $1/\tau_{el}$  and averaging over the Fermi line leads to the observed low- $T$  resistivity. To access the low- $T$  resistivity for the normal state a magnetic field perpendicular to the planes is applied, which remains unaltered down to  $T = 0$ , as in the cuprates [3]. The field serves merely to remove superconductivity and achieve the  $H$ -independent term  $\rho(T)$  of  $\rho(T, H) = \rho(T) + \delta\rho(T, H)$  where  $\delta\rho(T, H)$  is the magnetoresistance contribution. The  $T$ -dependent inverse re-

laxation lifetime derived by replacing  $\hbar\omega$  by  $\pi k_B T$  in the above one-electron inverse lifetime  $\hbar/\tau_{el} \approx \hbar\omega \pi \alpha_{\tau_{el}}$  and averaging over the Fermi line is given by,

$$\frac{1}{\tau(T)} = \frac{2}{\pi} \left( \int_0^{\pi/2} d\phi \frac{1}{\tau_{el}} \right) \Big|_{\hbar\omega = \pi k_B T} = \frac{1}{\hbar A} \pi k_B T, \quad (2)$$

$$A = \frac{32}{\pi^2} \sqrt{x x_{op}}, \quad x \in (x_A, x_{c2}).$$

The hole concentration  $x_A \approx x_*/2$  is that at which  $A \approx 0.5$  becomes of order one. The normal-state resistivity  $H$ -independent term  $\rho(T)$  of  $\rho(T, H)$  then reads,

$$\rho(T) \approx \left( \frac{m_c^p d_{\parallel}}{x e^2} \right) \frac{1}{\tau(T)}; \quad m_c^p = \frac{\hbar^2 \pi x_*}{2t}, \quad (3)$$

where  $m_c^p$  is the  $c$  fermion transport mass [10]. Combination of Eqs. (2) and (3) leads to the following resistivity expression,

$$\rho(T) \approx \left( \frac{\hbar d_{\parallel}}{t e^2} \right) \left( \frac{\pi}{4} \right)^3 \frac{x_*}{x^{3/2} \sqrt{x_{op}}} \pi k_B T. \quad (4)$$

Consistency with the above  $\hbar/\tau_{el}$  expression validity range  $\hbar\omega < E_1(\phi)$  implies that the behavior (4) remains dominant in the normal-state range  $T \in (0, T_1)$ . Here,

$$T_1 \approx \frac{2}{\pi} \int_0^{\pi/2} d\phi \frac{E_1(\phi)}{k_B} = \frac{(2\pi - 4)W_{ec}}{\pi^2 k_B}. \quad (5)$$

At  $x = 0.16$  this gives  $T_1 \approx 554$  K for LSCO and  $T_1 \approx 1107$  K for YBCO 123. Extrapolation of expression (4) to  $H = 0$  leads to  $\rho(T, 0) \approx \theta(T - T_c) \rho(T)$  for  $T < T_1$ . Here the critical low- $T$  resistivity behavior (4) is masked by the onset of superconductivity at  $T = T_c$ .

We now compare our theoretical linear- $T$  resistivity with that of LSCO [21] and YBCO 123 [22] for  $H = 0$  and  $T$  up to 300 K. Transport in the  $b$  direction has for YBCO 123 contributions from the CuO chains, which render our results unsuitable. In turn,  $\rho_a(T, 0) \approx \theta(T - T_c) \rho(T)$  at  $H = 0$  for the  $a$  direction.  $\rho(T, 0)$  and  $\rho_a(T, 0)$  are plotted in Figs. 2 and 3 for the parameters for LSCO and YBCO 123, respectively.  $x$  is between  $x \approx x_A \approx 0.135$  and  $x_{c2} \approx 0.20 - 0.22$  for the LSCO theoretical lines of Fig. 2. Fig. 3 for YBCO 123 refers to three  $x$  values near  $x_{op}$ , expressed in terms of the oxygen content. Comparison of the theoretical curves of Fig. 2 with the LSCO resistivity curves of Ref. [21] also shown in the figure confirms an excellent quantitative agreement between theory and experiments for the present range  $x \in (x_A, x_{c2})$ . In turn, for YBCO 123 our scheme provides a good quantitative description of the experimental curves near  $x_{op}$ , for  $y = 6.95, 7.00$ . The  $y = 6.85$  experimental curve of Ref. [22] already deviates from the linear- $T$  behavior. (The hole concentration that marks the onset of such a behavior reads for that material  $x_A \approx 0.15$ .)

For the present range  $x \in (x_A, x_{c2})$ , the interplay of the  $c$  Fermi line isotropy with the  $s1$  boundary line

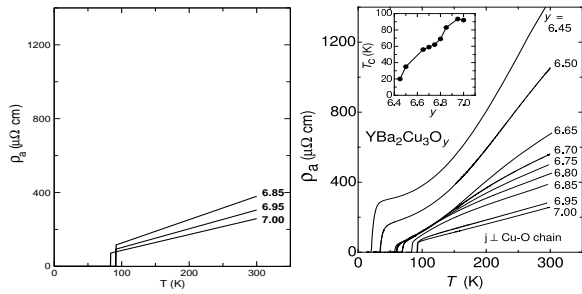


FIG. 3: (a) The  $T$  dependence of the resistivity  $\rho_a(T, 0) = \theta(T - T_c) \rho(T)$  where  $\rho(T)$  is given in Eq. (4) for the parameter magnitudes for YBCO 123 for a set of  $y$  values. (b) Corresponding experimental curves of Ref. [22]. The oxygen content  $y - 6$  is obtained from  $x$  by use of Fig. 4 (a) of Ref. [23]. The theoretical  $T_c$  is larger at  $y = 6.95$  than for  $y = 6.85, 7.00$ . This is alike in the inset of the second figure. Experimental curves figure from Ref. [22].

strong anisotropy [10, 11] plays an important role. It is behind the  $c$  -  $s_1$  fermion inelastic collisions leading to anisotropic one-electron scattering properties associated with the factor  $(\cos 2\phi)^2$  in the one-electron scattering rate expression. In turn, consistently with the experimental resistivity curves of Figs. 2 and 3, the non-linear  $T$  dependence of the resistivity developing for approximately  $x < x_A$  for a range of low temperatures that increases upon decreasing  $x$  is in part due to the matrix element  $W_{c,s1}(\vec{q}^h, \vec{q}; \vec{p})$  acquiring a different form due to the increase of the ratio  $r_\Delta = V_{Bs1}^\Delta / V_{Fc}$  magnitude. Our method does not apply to that regime. On the other hand, for the range  $x > x_{c2}$  also not considered here a competing scattering channel emerges, leading to an additional  $T^2$ -quadratic resistivity contribution [3, 4].

That the dependence on the Fermi angle  $\phi \in (0, \pi/2)$  of the scattering-rate coefficient  $\alpha = \alpha_{\tau_{el}} / \hbar V_F = (\cos 2\phi)^2 / (x 64 x_* \sqrt{\pi x_{op}} t)$  associated with that of the inverse lifetime  $\hbar / \tau_{el} \approx \hbar \omega \pi \alpha_{\tau_{el}}$  agrees with the experimental points of Fig. 1 seems to confirm the evidence provided in Refs. [10, 11] that the VEPQL may contain some of the main mechanisms behind the unusual properties of the hole-doped cuprates and their parent compounds. That in addition the inverse relaxation lifetime  $1/\tau$   $T$ -dependence obtained for  $\hbar \omega \ll \pi k_B T$  and  $x \in (x_A, x_{c2})$  by merely replacing  $\hbar \omega$  by  $\pi k_B T$  in  $1/\tau_{el}$  and averaging over the Fermi line leads to astonishing quantitative agreement with the resistivity experimental lines is a stronger surprising result. Consistently, for  $\hbar \omega \ll \pi k_B T$  and  $x \in (x_A, x_{c2})$  the system exhibits dynamics characterized by the relaxation time  $\tau = \hbar A / \pi k_B T$  of Eq. (2), where  $A = [32 \sqrt{x x_{op}} / \pi^2] \approx 1$  for  $x > x_A$ . This second stronger result provides clear evidence of normal-state scale-invariant physics. It may follow from beyond mean-field theory the  $T = 0$  line  $H_{c2}(x)$ , plotted in Fig. 4 of Ref. [11] for  $x \in (x_0, x_{c2})$ , referring to a true quantum phase transition. Such a

transition could occur between a state with long-range spin order regulated by monopoles and antimonopoles for  $H > H_{c2}$  and a vortex liquid by vortices and antivortices for  $H < H_{c2}$ . That would be a generalization for  $H_{c2} > 0$  of the quantum phase transition speculated to occur at  $x_{c2} \approx x_0$  and  $H_{c2} \approx 0$  in Ref. [8]. The field  $H^*$  marks a change from a short-range spin order to a disordered state and thus refers to a crossover. Hence the normal-state scale invariance occurring for  $x \in (x_A, x_{c2})$  could result from the hole concentration  $x_{c2} \approx 0.2$ , where the lines of Fig. 4 of Ref. [11] associated with the fields  $H_{c2}(x)$  and  $H^*(x)$  meet, referring to a quantum critical point [1, 2]. Such a quantum critical point may prevent the competing  $T^2$ -quadratic resistivity contribution to strengthen below  $x = x_{c2}$ .

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